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Discrete gauge symmetries and proton stability in the $\mathrm{U}(1)'$-extended MSSM

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Abstract: The Minimal Supersymmetric Standard Model (MSSM) with conserved R-parity suffers from several fine-tuning problems, e.g. the μ-problem and the problem of proton decay through higher dimension operators. Both of these problems can be avoided by replacing R-parity with a non-anomalous $\mathrm{U}(1)'$ gauge symmetry which is broken at the TeV scale. The new gauge symmetry does not necessarily forbid *all* renormalizable R-parity violating interactions among the MSSM fields, and may allow for either lepton number or baryon number violation at the renormalizable level. However, the proton decay problem resurfaces with the introduction of new TeV-scale exotic fields which are required for gauge anomaly cancellations. In this paper we investigate the issue of proton stability in the presence of TeV-scale exotics. We show that there are large classes of models in which TeV exotics do *not* destabilize the proton. We classify the viable models according to the residual discrete symmetries after $\mathrm{U}(1)'$ and electroweak symmetry breaking. In some of our examples the residual $\mathrm{U}(1)'$ discrete gauge symmetry within the MSSM sector alone ensures that the proton is absolutely stable, for *any* exotic representations. In other cases the proton can be sufficiently long-lived, depending on the $\mathrm{U}(1)'$ and hypercharge discrete charge assignments for the exotic fields. Our analysis outlines a general scheme for ensuring proton stability in the presence of light exotics with baryon and lepton number violating interactions.

Keywords: Discrete and Finite Symmetries, Beyond Standard Model, Supersymmetry Phenomenology.

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Contents

1. Introduction

In the Standard Model (SM) lepton number ($\mathcal{L}$) and baryon number ($\mathcal{B}$) are conserved at the renormalizable level due to accidental global symmetries. In the supersymmetric SM, with the addition of the superpartners, $\mathcal{L}$ and $\mathcal{B}$ are not conserved anymore. Therefore, the supersymmetrization of the SM requires an accompanying symmetry or some mechanism for ensuring proton stability. The Minimal version of the Supersymmetric Standard Model (MSSM) with R-parity has been the most popular model of low-energy supersymmetry. R-parity is a $\mathbb{Z}_2$ symmetry, which has been the prevailing candidate for the companion symmetry of supersymmetry as it protects the proton from decaying through *renormalizable* lepton number violating (LV) and baryon number violating (BV) terms.

However, R-parity alone does not completely cure the fine-tuning problems of the supersymmetric SM. First, R-parity still allows the existence of dangerous higher dimension operators (e.g. $QQQL$ and $U^cU^cD^cE^c$ in the superpotential), which violate both $\mathcal{L}$ and $\mathcal{B}$ and thus endanger proton stability [1 – 4].[1] This should be considered a serious flaw of the MSSM, given that R-parity was introduced to ensure proton stability in the first place. In addition, R-parity does not address the μ-problem [8] of the MSSM, whose solution may

[1] See, e.g. refs. [5 – 7], to see how the problem can be alleviated in grand unified theories.

require some other mechanism. These shortcomings of the MSSM motivate an alternative supersymmetrization of the SM and/or an alternative companion symmetry, especially since R-parity violation (RPV) leads to interesting phenomenology which is in agreement with all current experimental constraints [9 – 13].

The $\mathrm{U}(1)'$-extended MSSM (UMSSM) [14] is an extension of the MSSM with a new Abelian non-anomalous gauge symmetry $\mathrm{U}(1)'$ at the TeV scale.[2] In the UMSSM the μ-problem is solved by replacing the original μ term (H_2H_1) with an effective μ term (SH_2H_1) in the superpotential. Interestingly, it was recently found that the set of $\mathrm{U}(1)'$ charge assignments which solve the μ-problem, *automatically* forbid the coexistence of the renormalizable LV terms and BV terms, a phenomenon which was dubbed *LV-BV separation* [17]. Furthermore, the $\mathrm{U}(1)'$ symmetry also guarantees the absence of dangerous non-renormalizable proton decay operators constructed out of MSSM fields. Thus the $\mathrm{U}(1)'$ symmetry ties up the explanation of the proton longevity to the solution to the μ-problem and provides a solid theoretical framework for RPV phenomenology. We therefore find the R-parity violating UMSSM worth investigating as an alternative to the usual R-parity conserving MSSM.

However, the new gauge symmetry usually requires some exotic fields at the $\mathrm{U}(1)'$ breaking scale, in order to cancel the gauge anomalies [18 – 21]. Such light exotics would reintroduce the proton stability problem, since the exotics themselves may have LV and/or BV interactions, and may correspondingly mediate proton decay at unacceptable rates. Therefore, the argument for proton stability in the UMSSM needs to be extended to include the exotic representations. This discussion was postponed in ref. [17] and we shall complete it here. We shall systematically study the proton decay problem in the UMSSM by identifying the underlying discrete symmetries encoded in the set of phenomenologically viable $\mathrm{U}(1)'$ charge assignments. We shall then use the $\mathrm{U}(1)'$ discrete symmetry to argue that the proton is sufficiently stable even in the presence of light exotics. For simplicity we shall mostly concentrate on $\mathbb{Z}_3$ symmetries, although we shall consider more general $\mathbb{Z}_N$ examples as well.

Our UMSSM setup is reviewed in section 2, and in section 3 we identify the possible $\mathrm{U}(1)'$ discrete gauge symmetries $\mathbb{Z}_N$ among the MSSM fields only. Of special interest to us will be the three $\mathbb{Z}_3$ symmetries denoted as B_3, L_3 and M_3 (see section 3 for their exact definition). In the case of B_3, the $\mathrm{U}(1)'$ discrete gauge symmetry among the MSSM fields is already sufficient to argue for the *absolute* stability of the proton (see section 4). In case of L_3 and M_3, the $\mathrm{U}(1)'$ discrete symmetry needs to be extended to include the exotics fields as well (see section 5) and the analysis becomes more involved. Nevertheless, we still find various classes of models in which the proton lifetime is sufficiently long. Our argument is based on the combination of the $\mathrm{U}(1)'$ discrete gauge symmetry $\mathbb{Z}_N$ *and* the hypercharge discrete gauge symmetry $\mathbb{Z}_N^Y$ which is left over after electroweak symmetry breaking. In section 6, we identify all such "good" classes of models for the case of $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ discrete symmetries. The corresponding results for the $\mathbb{Z}_3^Y \times \mathbb{Z}_6$-type extensions are presented in

[2]Supersymmetric RPV models with an additional anomaly-free $\mathrm{U}(1)$ gauge symmetry have previously been considered in [15]. For anomalous $\mathrm{U}(1)$ approaches, see for example ref. [16] and references therein.

appendix A. Sections 7 and 8 provide explicit examples of anomaly-free $\mathrm{U}(1)'$ models. These serve the purpose of illustrating the successive steps which are necessary to check that the proton is sufficiently stable within a given model. Section 7 showcases all viable L_3 symmetric models with a $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ extended discrete symmetry. The L_3 symmetric models of ref. [17] with a $\mathbb{Z}_3^Y \times \mathbb{Z}_{N>3}$ extension are discussed in section 8, completing the proof of the claim made in ref. [17] that the proton is sufficiently stable in these models. In addition, we also present some examples of M_3 symmetric $\mathbb{Z}_3^Y \times \mathbb{Z}_{N>3}$-type charge assignments. Section 9 summarizes our results.

2. The Framework of the UMSSM

In this section we briefly review the framework of the $\mathrm{U}(1)'$-extended Minimal Supersymmetric Standard Model. We closely follow the conventions in ref. [17]. In order to break the $\mathrm{U}(1)'$ gauge symmetry spontaneously, we need to introduce a Higgs singlet S in addition to the MSSM fields. S is a singlet under the SM gauge group, but carries non-zero $\mathrm{U}(1)'$ charge. To solve the μ-problem, we require the $\mathrm{U}(1)'$ charges to be such that the original μ term is forbidden but an effective μ term is dynamically generated after S acquires a vacuum expectation value (vev) at the TeV scale. Accordingly, we require the superpotential term

$$W_\mu \;=\; hSH_2H_1 \;. \tag{2.1}$$

With regard to the Yukawa interactions, we assume

$$W_{\text{Yukawa}} \;=\; y^U_{jk}H_2Q_jU^c_k + y^D_{jk}H_1Q_jD^c_k + y^E_{jk}H_1L_jE^c_k + y^N_{jk}\left(\frac{S}{M}\right)^a H_2L_jN^c_k\,, \tag{2.2}$$

where we apply the standard notation for the MSSM superfields. The indices j, k label the three generations of quarks and leptons. Notice that we have also included Yukawa couplings for the right-handed neutrinos N^c. To explain the small neutrino mass in the case of pure Dirac neutrinos [22], we promote the Yukawa coefficient for the last term to have a suppression factor $\left(\frac{S}{M}\right)^a$ with a cutoff scale M and a positive-definite integer a.[3]

Assuming generation independence,[4] eqs. (2.1) and (2.2) yield five constraints on nine $\mathrm{U}(1)'$ charges. Denoting these charges by z, we have

$$Y_S \,:\, z[S] + z[H_1] + z[H_2] = 0\,, \tag{2.3}$$

$$Y_U \,:\, z[H_2] + z[Q] + z[U^c] = 0\,, \tag{2.4}$$

$$Y_D \,:\, z[H_1] + z[Q] + z[D^c] = 0\,, \tag{2.5}$$

$$Y_E \,:\, z[H_1] + z[L] + z[E^c] = 0\,, \tag{2.6}$$

$$Y_N \,:\, z[H_2] + z[L] + z[N^c] + az[S] = 0\;. \tag{2.7}$$

[3] The exact numerical value of M is not crucial for our discussion below. For example, M could be taken as high as the Planck scale, or as low as $\mathcal{O}(10^{15}$ GeV), as required for pure Dirac neutrino masses for $a = 1$ and $y^N \sim \mathcal{O}(1)$.

[4] Generation-dependent $\mathrm{U}(1)'$ charges may explain the discrepancies in rare B decays [23, 24].

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With the above relations, the $[\mathrm{SU}(3)_C]^2 - \mathrm{U}(1)'$ anomaly $A_{331'}$ cannot vanish unless we introduce additional colored particles. A minimal[5] and commonly considered extension of the particle spectrum is to assume three generations [17] of exotic quarks K_i, which are triplets under $\mathrm{SU}(3)_C$ and singlets under $\mathrm{SU}(2)_L$, as well as their right-handed counter-partners K_i^c. These exotic fields acquire their masses at the scale of $\mathrm{U}(1)'$ breaking from the superpotential terms

$$W_{\text{exotic}} = h_{ij} S K_i K_j^c \,, \tag{2.8}$$

which we assume to have non-vanishing diagonal couplings. Then, the $\mathrm{U}(1)_Y \times \mathrm{U}(1)'$ charges of the K_i^c are uniquely fixed by those of the K_i

$$y[K_i^c] = -\,y[K_i]\,, \qquad z[K_i^c] = -\,z[K_i] - z[S]\,. \tag{2.9}$$

While canceling $A_{331'}$, the exotic quarks introduce six new parameters. However, they do not affect the $[\mathrm{SU}(2)_L]^2 - \mathrm{U}(1)'$ anomaly $A_{221'}$

$$A_{221'}:\ 3(3z[Q] + z[L]) + N_H(z[H_1] + z[H_2]) = 0\,. \tag{2.10}$$

Eq. (2.10) yields another constraint on the $\mathrm{U}(1)'$ charges. Setting aside the exotic quarks for the moment, we can express the $\mathrm{U}(1)'$ charges of the remaining fields in terms of three free parameters α, β and γ:

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[N^c] \\ z[E^c] \\ z[H_1] \\ z[H_2] \\ z[S] \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 0 \\ 6 \\ -3 \\ 3 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ -1 \\ -3 \\ 3 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} N_H \\ 9 - N_H \\ -N_H \\ 0 \\ 9(1-a) \\ 0 \\ 0 \\ -9 \\ 9 \end{pmatrix}. \tag{2.11}$$

The first vector is proportional to the SM hypercharge, and the second corresponds to $\mathcal{B} - \mathcal{L}$. One can therefore use the freedom contained in the second and third vector to allow or forbid SM-invariant operators in the superpotential and the Kähler potential.

Since we do not a priori assume R-parity, the usual R-parity violating terms are allowed in general:

$$W_{\text{LV}} = \hat{\mu}_i L_i H_2 + \hat{\lambda}_{ijk} L_i L_j E_k^c + \hat{\lambda}'_{ijk} L_i Q_j D_k^c\,, \tag{2.12}$$
$$W_{\text{BV}} = \hat{\lambda}''_{ijk} U_i^c D_j^c D_k^c\,. \tag{2.13}$$

Motivated by the relatively tight experimental constraints on the individual RPV couplings $\hat{\mu}$, $\hat{\lambda}$, $\hat{\lambda}'$ and $\hat{\lambda}''$ [10], we shall exploit the possibility that they may originate from higher

[5]The only other alternative which would make $A_{331'}$ vanish, is an octet under $\mathrm{SU}(3)_C$ [18]. However, the choice of an octet is inconsistent with the remaining anomaly conditions, unless one is willing to unnecessarily complicate the exotic particle spectrum.

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dimension operators, and their values are suppressed by factors of $\left(\frac{\phi}{M}\right)^\ell$ where ϕ is a SM singlet combination of fields which acquires a vev, in our case S or H_2H_1. In general, we then have

$$\hat{\lambda} = \lambda \left(\frac{\langle S\rangle}{M}\right)^A \left(\frac{\langle H_2\rangle \langle H_1\rangle}{M^2}\right)^B , \tag{2.14}$$

(with positive-definite A and B) and similarly for $\hat{\mu}$, $\hat{\lambda}'$ and $\hat{\lambda}''$. For the sake of simplicity, in what follows we shall typically assume that $B = 0$, so that all suppression factors are of the type $\left(\frac{\langle S\rangle}{M}\right)^A$. This assumption is not crucial to our discussion, and only in sections 7 and 8 we shall revisit this issue, allowing for $\left(\frac{\langle H_2\rangle\langle H_1\rangle}{M^2}\right)^B$ type of suppression as well.

With those assumptions, the corresponding RPV superpotentials (2.12) and (2.13) become

$$W_{\rm LV} = h'_i \left(\frac{S}{M}\right)^n SL_iH_2 \;+\; \lambda_{ijk}\left(\frac{S}{M}\right)^n L_iL_jE^c_k \;+\; \lambda'_{ijk}\left(\frac{S}{M}\right)^n L_iQ_jD^c_k\,, \tag{2.15}$$

$$W_{\rm BV} = \lambda''_{ijk}\left(\frac{S}{M}\right)^m U^c_iD^c_jD^c_k\;. \tag{2.16}$$

Notice that all three LV terms in (2.15) have the same power of $1/M$ suppression, for which from now on we shall use n, while for the corresponding suppression in (2.16) we shall use m. The integers n and m should be considered among the input parameters of our UMSSM models.

Following ref. [17], we can easily demonstrate the LV-BV separation by taking the linear combination $6Y_D + 3Y_U - 3Y_E + (N_H - 3)Y_S - A_{221'}$, resulting in

$$3z[S^mU^cD^cD^c] - 3z[S^nLLE^c] + (N_H - 3(1 + m - n))z[S] = 0\;. \tag{2.17}$$

We see that the LV-BV separation observed in ref. [17] can now be generalized for any values of n and m. As long as the condition $N_H \neq 3\cdot\mathbb{Z}$ is kept, the third term in eq. (2.17) does not vanish and must be canceled by one (or the combination) of the first two terms. Notice that the first (second) term in eq. (2.17) is nothing but the $\mathrm{U}(1)'$ charge of the BV (LV) operator(s) in eq. (2.16) (eq. (2.15)). Therefore, the nonvanishing of the first or second term in eq. (2.17) implies that the corresponding renormalizable RPV couplings (BV or LV) are absent from the superpotential, as they are forbidden by the $\mathrm{U}(1)'$ gauge symmetry. The fact that certain terms are forbidden even at the non-renormalizable level (i.e. with arbitrary suppression factors $(S/M)^\ell$) suggests a certain symmetry. In section 3 we shall investigate the nature of the symmetry which is implied by the phenomenon of LV-BV separation. In what follows we shall restrict ourselves to the simplest and most natural case exhibiting LV-BV separation, namely $N_H = 1$.

The additional requirement of having either the LV terms LH_2, LLE^c, LQD^c of eq. (2.15) or the BV terms $U^cD^cD^c$ of eq. (2.16) at the effective level reduces the general solution eq. (2.11) to a two-parameter solution.

- For the LV case, one must demand that

$$z[H_1] \;=\; z[L] + nz[S]\,, \tag{2.18}$$

from eq. (2.15). Eq. (2.18) relates the parameters β and γ in eq. (2.11) by the condition $\beta = 3n\gamma$. The $\mathrm{U}(1)'$ charges in the LV case can then be written as

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[N^c] \\ z[E^c] \\ z[H_1] \\ z[H_2] \\ z[S] \end{pmatrix} = (\alpha + (3n + N_H)\gamma) \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 0 \\ 6 \\ -3 \\ 3 \\ 0 \end{pmatrix} + 3\gamma \begin{pmatrix} 0 \\ 3(1+n) + N_H \\ -3n - N_H \\ N_H \\ 3(1-a+n) \\ -3n - 2N_H \\ 3n + N_H \\ -3(1+n) - N_H \\ 3 \end{pmatrix} . \tag{2.19}$$

- In the BV case, we must require

$$z[H_1] = z[L] + \left(1 + m - \frac{N_H}{3}\right) z[S] \,, \tag{2.20}$$

from eq. (2.16). β and γ are now related by $\beta = (3 + 3m - N_H)\gamma$, and we obtain

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[N^c] \\ z[E^c] \\ z[H_1] \\ z[H_2] \\ z[S] \end{pmatrix} = (\alpha + 3(1+m)\gamma) \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 0 \\ 6 \\ -3 \\ 3 \\ 0 \end{pmatrix} + 3\gamma \begin{pmatrix} 0 \\ 3(2+m) \\ -3(1+m) \\ N_H \\ 3(2-a+m) - N_H \\ -3(1+m) - N_H \\ 3(1+m) \\ -3(2+m) \\ 3 \end{pmatrix} . \tag{2.21}$$

3. Discrete symmetries without exotics

Within the framework of the UMSSM, there are the usual MSSM particles, plus the right-handed neutrinos N_i^c, the Higgs singlet S and the exotic quarks K_i, $i = 1, 2, 3$. First, we want to discuss the occurrence of discrete symmetries at the *effective level*, where the K_i are integrated out. In that case, a general superpotential or Kähler potential operator has n_Q quark doublets, n_{U^c} up-type antiquark singlets, etc. If all $n_{\dots}$ are positive, the corresponding term appears in the superpotential. Negative $n_{\dots}$ are used for conjugate superfields, thus an operator with some $n_{\dots}$ being positive and others negative, can only appear in the Kähler potential. SM gauge invariance requires a certain relation among the $n_{\dots}$, see e.g. refs. [25, 26]. The total $\mathrm{U}(1)'$ charge of such a generic operator without exotic quarks is given as

$$\begin{aligned} z[\text{op.}] = {} & n_Q z[Q] + n_{U^c} z[U^c] + n_{D^c} z[D^c] + n_L z[L] + n_{N^c} z[N^c] + n_{E^c} z[E^c] \\ & + n_{H_1} z[H_1] + n_{H_2} z[H_2] + n_S z[S] \ . \end{aligned} \tag{3.1}$$

	$q[Q]$	$q[U^c]$	$q[D^c]$	$q[L]$	$q[N^c]$	$q[E^c]$	$q[H_1]$	$q[H_2]$
B_3	0	2	1	2	0	2	2	1
L_3	0	0	0	2	1	1	0	0
M_3	0	2	1	1	1	0	2	1
R_3	0	2	1	0	2	1	2	1

Table 1: The discrete anomaly-free $\mathbb{Z}_3$ symmetries defined on the MSSM+N^c sector.

The transition from the continuous $\mathrm{U}(1)'$ to its discrete subgroup $\mathbb{Z}_N$ is made by choosing a normalization in which the $\mathrm{U}(1)'$ charges are all integers, with

$$|z[S]| \equiv N\,, \tag{3.2}$$

and then defining the discrete charge $q[F] \in [0, N-1]$ of a field F by the mod N part of the corresponding $\mathrm{U}(1)'$ charge

$$z[F] = q[F] + N \cdot \mathbb{Z}\ . \tag{3.3}$$

Plugging this into eq. (3.1), we can define the discrete charge of any superpotential or Kähler potential operator by

$$\begin{aligned} q[\text{op.}] \equiv\ & n_Q q[Q] + n_{U^c} q[U^c] + n_{D^c} q[D^c] + n_L q[L] + n_{N^c} q[N^c] + n_{E^c} q[E^c] \\ & + n_{H_1} q[H_1] + n_{H_2} q[H_2]\ . \end{aligned} \tag{3.4}$$

The $\mathbb{Z}_N$ symmetry forbids any operator whose discrete charge $q[\text{op.}]$ is not a multiple of N. If it is a multiple of N, the operator might or might not exist, depending on the actual $\mathrm{U}(1)'$ charges. At this stage, we only want to argue from the discrete symmetry viewpoint.

The discrete anomaly-free $\mathbb{Z}_N$ symmetries of the MSSM fields with and without right-handed neutrinos N^c are known [27–30]. For $N = 3$, for example, all possibilities are shown in table 1. B_3 is the well-known baryon triality, and L_3 is correspondingly the lepton triality. R_3 is a symmetry related to the right-handed isospin. Finally, we have defined matter triality $M_3 \equiv R_3 L_3^2$. We point out that an overall sign flip of the discrete charges leads to exactly the same discrete symmetries. Such a sign change amounts to the exchange $1 \leftrightarrow 2$ in table 1.

We now extract the discrete symmetries which are encoded in the $\mathrm{U}(1)'$ charges of eqs. (2.19) and (2.21), i.e. the discrete symmetries for the LV case and the BV case. First, note that the hypercharge vector is irrelevant for the discussion of the remnant discrete symmetry of the $\mathrm{U}(1)'$ since the discrete symmetries are equivalent up to a shift by the hypercharges (normalized to integers). So it suffices to focus on the second vector (proportional to 3γ) of eqs. (2.19) and (2.21). In both cases, the minimum N is $N = z[S] = 3$ with $3\gamma = 1$. The discrete symmetries should be $\mathbb{Z}_3$ for the LV and the BV cases. More explicitly, we find

$$\begin{pmatrix} q[Q] \\ q[U^c] \\ q[D^c] \\ q[L] \\ q[N^c] \\ q[E^c] \\ q[H_1] \\ q[H_2] \end{pmatrix}_{\rm LV} = \begin{pmatrix} 0 \\ N_H \\ 2N_H \\ N_H \\ 0 \\ N_H \\ N_H \\ 2N_H \end{pmatrix}, \qquad \begin{pmatrix} q[Q] \\ q[U^c] \\ q[D^c] \\ q[L] \\ q[N^c] \\ q[E^c] \\ q[H_1] \\ q[H_2] \end{pmatrix}_{\rm BV} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ N_H \\ 2N_H \\ 2N_H \\ 0 \\ 0 \end{pmatrix}, \tag{3.5}$$

where we have used that $\mp N_H = \pm 2N_H$ mod 3. Note that these results do not depend on the specific values for n or m. Comparing with table 1 shows that, for $N_H \neq 0$ mod 3, the LV case yields baryon triality B_3 whereas the BV case gives rise to lepton triality L_3. Therefore, the origin of the LV-BV separation exhibited by eq. (2.17) can now be traced back to the existence of the discrete symmetries L_3 and B_3.

So far, we have investigated the discrete symmetries encoded in the UMSSM which solves the μ-problem. We identified B_3 in the LV case (where the LH_2, LLE^c and LQD^c terms are effectively present), and L_3 in the BV case (where the $U^cD^cD^c$ term effectively appears). Of course, one does not have to require any of these lepton or baryon number violating interactions. Then one can end up with other discrete symmetries as well. To see this, we rewrite eq. (2.11) by constructing a new basis of the three-parameter solution in which the first component of both the second and the third vector is zero. With $\alpha' = \alpha + \beta + N_H\gamma$, $\beta' = 3\beta$, $\gamma' = 3\gamma$, we get

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[N^c] \\ z[E^c] \\ z[H_1] \\ z[H_2] \\ z[S] \end{pmatrix} = \alpha' \begin{pmatrix} 1 \\ -4 \\ 2 \\ -3 \\ 0 \\ 6 \\ -3 \\ 3 \\ 0 \end{pmatrix} + \beta' \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 1 \\ -1 \\ 1 \\ -1 \\ 0 \end{pmatrix} + \gamma' \begin{pmatrix} 0 \\ 3+N_H \\ -N_H \\ N_H \\ 3(1-a) \\ -2N_H \\ N_H \\ -3-N_H \\ 3 \end{pmatrix}. \tag{3.6}$$

Assuming $N_H = 1$, we obtain matter triality M_3 as the remnant discrete symmetry in models where $\beta' = 1 \bmod 3$ and $\gamma' = 1$ (or $\beta' = -1 \bmod 3$ and $\gamma' = -1$). It is worth pointing out that the charge assignment of R_3 has $q[L] = 0$, which, due to our previous assumption $N_H \neq 0$ mod 3, can only be obtained for $\gamma' = 0$. This, however, is inconsistent, as the Higgs singlet S would then be neutral under $\mathrm{U}(1)'$. For $N_H = 1$ there are thus only three $\mathbb{Z}_3$ symmetries which can be generated from eq. (3.6):

$$\begin{aligned} B_3: &\quad \beta' = 0 \text{ mod } 3\,, \quad \gamma' = 1\,, \\ M_3: &\quad \beta' = 1 \text{ mod } 3\,, \quad \gamma' = 1\,, \\ L_3: &\quad \beta' = 2 \text{ mod } 3\,, \quad \gamma' = 1\,. \end{aligned} \tag{3.7}$$

	operators with $\mathcal{B}$ violation	operators with $\mathcal{L}$ violation
B_3	none	N^c; $LH_2,\ N^cN^c,\ N^cS$; $LQD^c,\ LLE,\ SLH_2,\ N^cH_1H_2,\ LH_1^\dagger$, $N^cN^cN^c,\ N^cN^cS,\ N^cSS,\ N^cS^\dagger$; many dimension five terms
L_3	$U^cD^cD^c$; $QQQH_1,\ SU^cD^cD^c,\ QQD^{c\dagger}$	$N^cN^cN^c$; $SN^cN^cN^c$
M_3	none	$N^cN^cN^c$; $SN^cN^cN^c$

Table 2: $\mathcal{B}$ and/or $\mathcal{L}$ violating operators up to dimension five which conserve a $\mathbb{Z}_3$ symmetry and comprise only MSSM particles, right-handed neutrinos N^c and Higgs singlets S.

With $\gamma' = -1$, the above β' would also have to flip sign in order to yield the same discrete symmetries. Choosing $\gamma' \neq \pm 1$ generically leads to $\mathbb{Z}_N$ symmetries with higher N. However, notice that the LV case (i.e. with LLE^c etc.) and the BV case ($U^cD^cD^c$) requirement leads only to B_3 and L_3 or their simple scaling (such as B_6^2, L_6^2), respectively.

The discrete $\mathbb{Z}_N$ symmetries encoded in the $\mathrm{U}(1)'$ charges provide a powerful tool to see which $\mathcal{L}$ and/or $\mathcal{B}$ violating operators could in principle arise in the theory. All such operators up to dimension five are summarized in table 2 for the three possible $\mathbb{Z}_3$ symmetries in eq. (3.7). As table 2 demonstrates, B_3 allows a number of LV terms but does not allow the BV terms of eq. (2.16), in accord with LV-BV separation. Similarly, L_3 allows BV terms but does not allow the LV terms of eq. (2.15). Finally, M_3 forbids both the LV terms of eq. (2.15) and the BV terms of eq. (2.16).

Since proton decay requires both baryon number violation as well as lepton number violation, the absence of either one of them is sufficient to stabilize the proton. Moreover, any $\mathcal{B}$ and/or $\mathcal{L}$ violating interaction which is suppressed by two powers of a high cutoff scale M does not endanger the proton. That is why table 2 lists only superpotential and Kähler potential operators up to dimension five. B_3 and M_3 conserve $\mathcal{B}$ up to this level, while L_3 and M_3 have $\mathcal{L}$ violation only through the two operators $N^cN^cN^c$ and $SN^cN^cN^c$, which could be forbidden by a judicious choice of the $\mathrm{U}(1)'$ charges $z[N^c]$ and $z[S]$. Table 2 reveals that in models exhibiting a B_3 or M_3 discrete symmetry, the proton cannot be destabilized by *any* diagram involving MSSM fields, N^c and S. In models with an L_3 symmetry, it is simply sufficient that one forbids the $N^cN^cN^c$ and $SN^cN^cN^c$ superpotential terms, and the proton is safe from such diagrams as well.

While these statements are true to all orders in perturbation theory, they are of limited use due to an important caveat which we must take into account. So far in this section we have ignored the effect of the exotics. Once we take them into account, the scale which suppresses the non-renormalizable operators in table 2 is by far not guaranteed to be the high scale M. In fact, the exotics are relatively light, near the TeV scale, since they get their masses from the $\mathrm{U}(1)'$ breaking scale (see eq. (2.8)). Thus, depending on their couplings

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discrete symmetry	$q[Q]$	$q[U^c]$	$q[D^c]$	$q[L]$	$q[N^c]$	$q[E^c]$	$q[H_1]$	$q[H_2]$
$B_3 \to \mathbb{Z}_9$	-1	1	1	0	0	0	0	0
$\mathbb{Z}_T$	t_Q	t_{U^c}	t_{D^c}	t_L	t_{N^c}	t_{E^c}	t_{H_1}	t_{H_2}
$\mathbb{Z}_9 \times \mathbb{Z}_T$	$-T+9t_Q$	$T+9t_{U^c}$	$T+9t_{D^c}$	$9t_L$	$9t_{N^c}$	$9t_{E^c}$	$9t_{H_1}$	$9t_{H_2}$

Table 3: The discrete charges of the B_3 equivalent $\mathbb{Z}_9$ and $\mathbb{Z}_9 \times \mathbb{Z}_T$.

to the MSSM+N^c sector, the exotics could in principle destabilize the proton.

Before delving into the details of how to take account of the exotic quarks, we shall take a detour to explain why certain $\mathbb{Z}_N$ symmetries such as baryon triality B_3 or proton hexality P_6 [29] are actually sufficient to *completely* stabilize the proton, independent of the presence of any light exotics or their properties.

4. The absolute stability of the proton

We have just emphasized that proton decay requires violation of both $\mathcal{B}$ and $\mathcal{L}$. However, not any type of $\mathcal{B}$ violation can lead to a decaying proton. In order to understand this, first observe that the exotic quarks are heavier than the proton (otherwise they would have been produced and seen at colliders). Hence, for kinematic reasons, the proton cannot decay to exotic quarks in the final state, and we can exclude the exotics K_i from the discussion in this section. Since the proton is the lightest particle with non-zero baryon number ($\mathcal{B}=1$), the final state particles must be non-baryonic. Therefore, *proton decay requires an interaction which has* $|\Delta\mathcal{B}|=1$. A theory where $\mathcal{B}$ is only violated by a certain number of units $\eta > 1$ (so that *any* BV operator in the theory has $|\Delta\mathcal{B}| = 0 \bmod \eta$), automatically has an absolutely stable proton. This was observed in ref. [31] for the specific case of baryon triality B_3.

Here, we first review the argument for B_3, with the sign convention adopted in table 1. Since it is a $\mathbb{Z}_3$ symmetry, the two Higgs doublets have opposite discrete charges $-q[H_1] = q[H_2] = 1 \bmod 3$. It is possible to redefine the $q[F]$ by adding a certain amount (take e.g. $\alpha' = -\frac{1}{3}$) of the hypercharge vector in eq. (3.6), so that $q[H_1] = q[H_2] = 0 \bmod 3$. As discrete charges should always be integer, it is necessary to rescale the resulting vector by multiplication of 3. We obtain the $\mathbb{Z}_9$ charge assignment, which is exactly $-3\mathcal{B}$, as shown in the first row of table 3. Then the total discrete charge of any operator is

$$q[\text{op.}] = -(n_Q - n_{U^c} - n_{D^c}) = 0 \bmod 9\,, \tag{4.1}$$

while for proton decay we need $|\Delta\mathcal{B}|=1$, i.e.

$$n_Q - n_{U^c} - n_{D^c} = \pm 3\,, \tag{4.2}$$

which is incompatible with eq. (4.1). Thus, the proton is absolutely stable if the discrete symmetry is B_3.

This argument can be applied to more general cases. If our U(1)′ has a discrete symmetry of $B_3 \times \mathbb{Z}_T$ (with T coprime to 3), then the proton is also absolutely stable.[6] The resultant discrete charge of $\mathbb{Z}_{9T}$ is given by $Tq_{Z_9} + 9q_{Z_T}$ as table 3 shows.

There are examples of this kind in ref. [30] where all anomaly-free $\mathbb{Z}_{N\leq 14}$ symmetries were identified:[7] $R_6L_6^4$ (proton hexality or P_6 [29]), $R_{12}L_{12}^4$ and $R_{12}^5L_{12}^8$. These three symmetries are isomorphic to the direct product of B_3 with some other $\mathbb{Z}_T$ symmetries and provide absolute proton stability

$$R_6L_6^4 \cong B_3 \times R_2\,, \tag{4.3}$$
$$R_{12}L_{12}^4 \cong B_3 \times R_4^3\,, \tag{4.4}$$
$$R_{12}^5L_{12}^8 \cong B_3 \times R_4\,. \tag{4.5}$$

5. Discrete symmetries with exotic quarks

In table 2, we have listed only $\mathcal{B}$ and/or $\mathcal{L}$ violating operators up to dimension five. This is sufficient to argue for a stable proton only under the assumption that the non-renormalizable interactions between the particles are generated by high scale physics. However, since the exotic quarks were integrated out, the mass suppression of the non-renormalizable terms could in principle be of $\mathcal{O}$(TeV), the scale where the U(1)′ breaks down and gives mass to the K_i. In such a case, one should also consider $\mathcal{B}$ and/or $\mathcal{L}$ violating operators with dimensionality higher than five.

Alternatively, one can try to extend the $\mathbb{Z}_N$ symmetries to *explicitly include the exotic particles*. Under the assumption that any additional new physics (other than U(1)′) occurs only at the high scale, the non-renormalizable interactions involving the MSSM particles, the right-handed neutrinos, the Higgs singlet as well as the exotic quarks are all suppressed by powers of M. Thus the advantage of this approach is that we still only need to consider a finite number of operators (up to dimension 5), albeit over an extended set of fields. Similarly to the case without exotic quarks, the obtained *discrete* symmetries can be studied with respect to the allowed $\mathcal{B}$ and/or $\mathcal{L}$ violating operators, where by definition the exotic quarks and antiquarks do not carry baryon or lepton number.

We now have to assign discrete charges to the three exotic quarks K_i. The corresponding antiquarks K_i^c automatically have the opposite discrete charges. Assuming that their original U(1)′ charges are integers, we have 27 different $\mathbb{Z}_3$ charge assignments for the exotics K_i. However, as their generations have not been defined yet, we need to consider only 10 of these 27 possibilities, see table 4. When determining the allowed $\mathcal{B}$ and/or $\mathcal{L}$ violating superpotential and Kähler potential operators in section 3, we have required invariance under the $\mathbb{Z}_N$ subgroup of U(1)′, but also — tentatively — SM gauge invariance. We do not know the hypercharges of the K_i, and in principle there could be infinitely many hypercharge assignments. In order to systematize this issue, we work in a normalization in which $y[Q] = 1$ and assume that the hypercharges of the exotic quarks are integers. Now,

[6] The direct product of two cyclic groups ($\mathbb{Z}_A \times \mathbb{Z}_B$) is a cyclic group $\mathbb{Z}_{AB}$ if A and B have greatest common divisor 1 (i.e. if they are coprime).

[7] See ref. [28] for the definition of the generators R_N and L_N for arbitrary values of N.

$q[K_1]$	0	1	2	0	0	0	1	0	1	0
$q[K_2]$	0	1	2	0	0	1	1	2	2	1
$q[K_3]$	0	1	2	1	2	1	2	2	2	2

Table 4: 10 of 27 possible $\mathbb{Z}_3$ charge assignments for the exotic quarks are sufficient since the labeling of the generation for the exotics has not been determined yet.

we can require invariance under any $\mathbb{Z}_N^Y$ subgroup of $\mathrm{U}(1)_Y$, with the discrete hypercharges q^Y being defined by the relation

$$y[F] = q^Y[F] + N \cdot \mathbb{Z} \ . \tag{5.1}$$

For simplicity, we choose[8] $N = 3$, so that we end up with 27 discrete hypercharge assignments for the exotic quarks.

Starting from an $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'$ gauge theory, we study only its subgroup $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathbb{Z}_3^Y \times \mathbb{Z}_3$. The discrete charges of the MSSM particles are uniquely defined once we have picked our preferred $\mathbb{Z}_3$ symmetry among the MSSM fields. Due to the different $\mathrm{U}(1)_Y \times \mathrm{U}(1)'$ charge assignments for the exotic quarks, we have $10 \times 27 = 270$ cases to study. Each scenario is defined by the discrete hypercharges $q^Y[K_i]$ and the discrete $\mathrm{U}(1)'$ charges $q[K_i]$. We determine all $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathbb{Z}_3^Y \times \mathbb{Z}_3$ invariant operators up to dimension five and check whether there exists a conserved quantity $\mathcal{Q}$ of the form

$$\mathcal{Q} \ \equiv \ b\,\mathcal{B} + \ell\,\mathcal{L} + k_1\,\mathcal{K}_1 + k_2\,\mathcal{K}_2 + k_3\,\mathcal{K}_3 \,, \tag{5.2}$$

with b, ℓ, k_i being integers and $\mathcal{K}_i$ denoting (individual) exotic quark number, respectively. We stress that $\mathcal{Q}$ is conserved only among the operators up to dimension five. However, since any operator that is suppressed by two powers of M is not dangerous for proton decay, we loosely speak of "$\mathcal{Q}$ conservation" in the following, mindful of its approximate meaning.

Let us illustrate the implications of such a quantity $\mathcal{Q}$ with two examples. First, assume that the set of allowed operators up to dimension five has $\mathcal{Q}_1 = \mathcal{B}$ (e.g. as model (v) of table 8). Then, among these operators, baryon number is conserved and the proton is sufficiently stabilized. Next, we take $\mathcal{Q}_2 = 3\mathcal{L} - \mathcal{K}_1 + 2\mathcal{K}_2 + 2\mathcal{K}_3$ (e.g. as model (i) of table 8). In this case, both baryon and lepton number are violated. Concerning proton decay, however, the exotic quarks K_i are heavier than the proton and therefore cannot be present among the final state particles. So any diagram that potentially mediates proton decay necessarily has $\mathcal{K}_i = 0$. Due to the conservation of $\mathcal{Q}_2$, lepton number is conserved in all such diagrams, again leading to a stable proton. Although baryon and lepton number are both violated in the second example, the proton does not decay rapidly. We emphasize that this reasoning does not depend on whether one considers a specific model with fixed $\mathrm{U}(1)_Y \times \mathrm{U}(1)'$ charges or a scenario which imposes only a discrete subgroup $\mathbb{Z}_3^Y \times \mathbb{Z}_3$.

[8]After electroweak symmetry breaking (EWSB), the choice $N = 3$ coincides with the *remnant* discrete symmetry of $\mathrm{U}(1)_Y$, as opposed to the $\mathbb{Z}_N^Y$ symmetries with arbitrary N which exist before EWSB.

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$q^Y[Q]$	$q^Y[U^c]$	$q^Y[D^c]$	$q^Y[L]$	$q^Y[N^c]$	$q^Y[E^c]$	$q^Y[H_1]$	$q^Y[H_2]$	$q^Y[K_i]$	$q^Y[K_i^c]$
1	-1	-1	0	0	0	0	0	$q^Y[K]$	$-q^Y[K]$

Table 5: The generation independent $\mathbb{Z}_3^Y$ charges.

To find $\mathcal{Q}$, we have to solve a homogeneous set of J linear equations, each equation corresponding to one allowed operator. Denoting the baryon number of operator j by $\mathcal{B}[j]$ and likewise for the lepton and the exotic quark number, we are looking for coefficients (b, ℓ, k_1, k_2, k_3) which satisfy

$$b\,\mathcal{B}[j] + \ell\,\mathcal{L}[j] + k_1\,\mathcal{K}_1[j] + k_2\,\mathcal{K}_2[j] + k_3\,\mathcal{K}_3[j] \;=\; 0\,, \tag{5.3}$$

for all $1 \leq j \leq J$. Having J equations, at most five of them can be linearly independent. The number of linearly independent equations is called the rank $\mathfrak{r}$ of the set of equations. In the case where $\mathfrak{r} = 5$, the only solution to eq. (5.3) is $b = \ell = k_1 = k_2 = k_3 = 0$, thus no conserved quantity $\mathcal{Q}$ exists. If, however, $\mathfrak{r} < 5$, a non-trivial solution exists and with it a conserved quantity $\mathcal{Q}$ is guaranteed.

In the following section we fix the $\mathbb{Z}_3$ symmetry among the MSSM particles and scan over all 270 possible extensions of $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ which include the exotic quarks. For each case, we determine the allowed operators and calculate the rank $\mathfrak{r}$, keeping only those cases with $\mathfrak{r} < 5$.

6. Good $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ extensions

Here we do not need to consider the case of B_3 since it already guarantees absolute proton stability as discussed in section 4. We will therefore only consider extensions of M_3 and L_3 in this section.

M_3 extensions. Out of the 270 possible discrete charge assignments, only 20 lead to a reduced rank. Interestingly, it is the discrete hypercharge $\mathbb{Z}_3^Y$ that is responsible for the occurrence of a non-trivial conserved quantity $\mathcal{Q}$. This arises if

$$q^Y[K_1] \;=\; q^Y[K_2] \;=\; q^Y[K_3] \;=\; q^Y[K] \;=\; 0 \text{ or } 2\,, \tag{6.1}$$

regardless of the charges $q[K_i]$ under $\mathbb{Z}_3$. In these scenarios, $\mathcal{Q} = \mathcal{B}$, so baryon number is conserved up to dimension five operators including the exotic quarks. Therefore, the proton is sufficiently stable.

It is instructive to figure out the reason for this peculiar result. Under M_3, operators composed of only MSSM particles and right-handed neutrinos do not violate baryon number (see table 2). We will show that the inclusion of exotic quarks does not allow the construction of $\mathcal{B}$ violating operators up to dimension five which at the same time conserve the generation independent discrete hypercharge $\mathbb{Z}_3^Y$ displayed in table 5. Invariance under $\mathbb{Z}_3^Y$ requires

$$n_Q - n_{U^c} - n_{D^c} + q^Y[K](n_K - n_{K^c}) \;=\; 0 \bmod 3\;. \tag{6.2}$$

On the other hand, $\mathrm{SU}(3)_C$ invariance demands

$$n_Q - n_{U^c} - n_{D^c} + n_K - n_{K^c} \;=\; 0 \bmod 3\;. \tag{6.3}$$

Subtracting eq. (6.3) from eq. (6.2) yields

$$(q^Y[K]-1)(n_K - n_{K^c}) \;=\; 0 \bmod 3\,, \tag{6.4}$$

which for $q^Y[K] = 0$ or 2 can only be satisfied if

$$(n_K - n_{K^c}) = 0 \bmod 3\;. \tag{6.5}$$

Then, eq. (6.3) simplifies to

$$n_Q - n_{U^c} - n_{D^c} \;=\; 0 \bmod 3\,, \tag{6.6}$$

showing that baryon number violation requires at least three baryonic fields. Without exotic particles, the symmetry M_3 ensures that no baryon number violation occurs up to dimension five. Allowing for the presence of exotic quarks in such an operator, we would need at least two of them because of eq. (6.5). An operator including three baryons (in order to have baryon number violation) and two exotic quarks is, however, suppressed by at least two powers of M. Hence, it is the invariance under $\mathrm{SU}(3)_C$ and $\mathbb{Z}_3^Y$ that is responsible for baryon number conservation in M_3 extensions with $q^Y[K] = 0$ or 2.

It turns out that the same holds true for all $\mathbb{Z}_N$ symmetries that have $\mathcal{B}$ conservation up to dimension five among the MSSM particles and the right-handed neutrinos, for instance the $\mathbb{Z}_6$ symmetries $R_6L_6^2$ ($\cong M_3\times R_2$), and $R_6^3L_6^2$ ($\cong L_3\times R_2$) [29, 30] (see also appendix A).

L_3 extensions. Scanning over the $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ extensions of L_3 shows that $\mathfrak{r} = 5$ always. Therefore, with only the discrete symmetry at our disposal, we do not obtain a conserved quantity $\mathcal{Q}$. However, since L_3, to some extent, suggests the conservation of lepton number, one could remove the L_3 invariant but lepton number violating operators $N^cN^cN^c$ and $SN^cN^cN^c$ from the set of allowed operators (see table 2), and determine $\mathfrak{r}$ for the remaining sets. The idea behind this procedure is that one can easily forbid these two interactions with the underlying $\mathrm{U}(1)'$ by demanding $z[N^c] \neq 0$ and $z[N^c] \neq -z[S]/3$. Disregarding these operators, we find that the rank is reduced to four in 55 of the 270 possible extensions. The conserved quantity among the remaining operators is always $\widetilde{\mathcal{Q}} = \mathcal{L}$. Not all of these discrete charge assignments are compatible with the $[\mathrm{U}(1)_Y]^2 - \mathrm{U}(1)'$ anomaly condition $A_{111'}$, i.e. eq. (42) in ref. [17]. With $N_H = 1$ and the normalization where $y[Q] = 1$, $A_{111'}$ translates to

$$\sum_{i=1}^{3} y[K_i]^2 \;=\; 36\;. \tag{6.7}$$

The only integer solutions are $(0, 0, \sigma_3\cdot 6)$ and $(\sigma_1\cdot 2, \sigma_2\cdot 4, \sigma_3\cdot 4)$, with $\sigma_i = \pm 1$, as well as permutations thereof. Translated to the discrete hypercharges, these solutions correspond to $(0, 0, 0)$ and $(\sigma_1\cdot 2, \sigma_2\cdot 1, \sigma_3\cdot 1)$. With the implicit convention that $-2 = 1$ and $-1 = 2$, all viable discrete hypercharges are therefore of the form

$$\Big(q^Y[K_1], q^Y[K_2], q^Y[K_3]\Big) \;=\; (0,0,0) \;\;\text{or}\;\; (\sigma_1\cdot 1, \sigma_2\cdot 1, \sigma_3\cdot 1)\;. \tag{6.8}$$

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$\mathbb{Z}_3^Y$ \ $\mathbb{Z}_3$	$(0,0,0)$	$(1,1,1)$	$(2,2,2)$	$(0,0,1)$	$(0,0,2)$	$(0,1,1)$	$(1,1,2)$	$(0,2,2)$	$(1,2,2)$	$(0,1,2)$
$(0,0,0)$	✓ ☺	✓ ☺	✓ ☺							
$(1,1,1)$	✓									
$(2,2,2)$	✓ ☺	✓ ☺	✓ ☺							
$(1,1,2)$	✓			✓	✓					
$(1,2,1)$	✓									
$(2,1,1)$	✓									
$(1,2,2)$	✓					✓		✓		
$(2,1,2)$	✓									
$(2,2,1)$	✓									

Table 6: The $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ extensions of L_3. The discrete hypercharges $q^Y[K_i]$ that satisfy eq. (6.8) are shown in the rows; the discrete charges $q[K_i]$ of table 4 are given in the columns. The symbol ✓ indicates that lepton number is violated only in the operators $N^cN^cN^c$ and $SN^cN^cN^c$. A smiley ☺ denotes cases where baryon number is only violated in $U^cD^cD^c$, $SU^cD^cD^c$, $QQQH_1$ and $QQD^{c\dagger}$.

Out of the 55 $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ extensions of L_3 which reduce the rank, only 17 cases comply with eq. (6.8). They are listed in table 6. The symbol ✓ indicates the 17 cases in which $\widetilde{\mathcal{Q}} = \mathcal{L}$, i.e. those cases in which lepton number can only be violated by $N^cN^cN^c$ and $SN^cN^cN^c$. For those 6 symmetries additionally marked with the symbol ☺, the only baryon number violating operators up to dimension five are $U^cD^cD^c$, $SU^cD^cD^c$, $QQQH_1$ and $QQD^{c\dagger}$, neither of which involves exotic fields.

$$\checkmark: \quad \mathcal{L} \text{ violation only in } N^cN^cN^c,\ SN^cN^cN^c\ .$$
$$☺: \quad \mathcal{B} \text{ violation only in } U^cD^cD^c,\ SU^cD^cD^c,\ QQQH_1,\ QQD^{c\dagger}\ .$$

The remaining $17-6=11$ cases violate baryon number also in many interactions involving exotic quarks.

For the symmetries in table 6 indicated by ✓, the proton can be stabilized by forbidding the two lepton number violating operators by the continuous $\mathrm{U}(1)'$. In the cases marked with the symbol ☺, one could alternatively control the four baryon number violating interactions; if, for instance, $N^cN^cN^c$ is absent but $SN^cN^cN^c$ is allowed, one just has to forbid the renormalizable term $U^cD^cD^c$ with the $\mathrm{U}(1)'$ in order to make the proton sufficiently stable.

7. L_3 symmetric $\mathrm{U}(1)'$ models

In this section, we present phenomenologically viable and anomaly-free $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'$ models which have a $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ extension of L_3 as a subgroup. We choose $N_H = 1$, $a = 1$ and $m = 0$ or $m = -1$.[9] Additionally, we showcase two anomaly-free models which are incompatible with either proton longevity or the measured quark masses.

[9] Recall that up to now we have been assuming $n, m \geq 0$, so that any $1/M$ suppression in the dimensionless couplings is coming solely from $\frac{S}{M}$ factors. However, it can be readily seen from eqs. (2.1) and (2.14) that negative values of n and m are also possible, and could be interpreted as a corresponding suppression due to $\frac{H_2 H_1}{M^2}$ factors instead.

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Requiring $\mathrm{U}(1)_Y \supset \mathbb{Z}_3^Y$ entails integer hypercharges for all particles, including the exotic quarks. Therefore eq. (6.7) has only a finite number of solutions. In our search for concrete models, we choose the following 8+2 assignments

$$(y[K_1], y[K_2], y[K_3]) \;=\; \begin{cases} (\sigma_1\!\cdot\!2, \sigma_2\!\cdot\!4, \sigma_3\!\cdot\!4)\,, \\ (0, 0, \sigma_3\!\cdot\!6)\,, \end{cases} \tag{7.1}$$

with $\sigma_i = \pm 1$. All other possibilities are obtained from these by relabeling the generations of the exotic quarks.

With regard to the $\mathrm{U}(1)'$ symmetry, eq. (2.21) shows that one particular charge assignment is accompanied by a two-dimensional space of solutions which allow and forbid exactly the same operators.[10] One dimension is spanned by adding a certain amount of the hypercharge vector to the original charge assignment, the other arises due to the choice of the overall normalization. To be explicit, we keep only those assignments with $z[Q] = 0$ and $|z[S]| = 3$; the overall sign is fixed by demanding compatibility of eq. (3.3) with table 1. In other words, we take $\alpha + 3(1+m)\gamma = 0$ and $3\gamma = -1$ in eq. (2.21). In order to end up with a $\mathbb{Z}_3$ symmetry after $\mathrm{U}(1)'$ breaking, the charges $z[K_i]$ must be integers. As the cubic anomaly $[\mathrm{U}(1)']^3$ is quadratic in $z[K_i]$, we need to scan only over a finite number of assignments $(z[K_1], z[K_2], z[K_3])$ to find all anomaly-free models. The phenomenologically viable models are listed in table 7. Comparing with table 6 shows that models I–III and VI–VIII belong to the class "✓ ☺" with a $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ symmetry that allows only a few baryon and lepton number operators. Their absence or presence in the specific model can be checked immediately. We find:

- **I–III:** Neither $N^cN^cN^c$ nor $SN^cN^cN^c$ is allowed by the $\mathrm{U}(1)'$, so lepton number is conserved up to dimension five. The only baryon number violating operator is $U^cD^cD^c$. Therefore, the proton is safe in these models. Up to a hypercharge shift and an overall minus sign, model I is identical to the "BV–I" case in ref. [17].
- **VI–VIII:** Here, lepton and baryon number are separately violated, but only in non-renormalizable operators, namely $SN^cN^cN^c$, $QQQH_1$ and $QQD^{c\dagger}$. Hence, any diagram that makes the proton decay is necessarily suppressed by at least two powers of M, leading to a sufficiently long proton lifetime. Model VI is equivalent to the "BV–IV" case in ref. [17].

In order to see that the proton does not decay rapidly in the remaining $14 - 6 = 8$ cases of table 7, we need to construct all allowed operators up to dimension five and determine the conserved quantity $\mathcal{Q}$ for each model individually. We obtain:

- **IV:** $\mathcal{Q} = \mathcal{L} - \mathcal{K}_3$. Baryon number is therefore violated (through $U^cD^cD^c$), but lepton number is conserved in processes where there is no external exotic quark. Thus the proton is sufficiently stable.
- **V:** $\mathcal{Q} = \mathcal{L} + \mathcal{K}_3$. Same as for model IV.

[10] Note, however, that two models with different $\mathrm{U}(1)'$ charge assignments have different couplings to the Z' and $\tilde{Z}'$.

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	L_3 models with $m=0$					L_3 models with $m=-1$								
$z[Q]$	0					0								
$z[U^c]$	-6					-3								
$z[D^c]$	3					0								
$z[L]$	-1					-1								
$z[N^c]$	-2					1								
$z[E^c]$	4					1								
$z[H_1]$	-3					0								
$z[H_2]$	6					3								
$z[S]$	-3					-3								
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
in ref. [16]	BV-I					BV-IV								
$z[K_1]$	3	2	1	1	2	3	1	2	2	1	2	2	1	1
$z[K_2]$	-3	2	1	1	2	0	1	2	2	1	1	1	2	2
$z[K_3]$	-3	8	-5	8	-5	0	4	-1	4	-1	4	-1	4	-1
$y[K_1]$	2	0	0	0	0	2	0	0	0	0	0	0	0	0
$y[K_2]$	-4	0	0	0	0	-4	0	0	0	0	0	0	0	0
$y[K_3]$	-4	6	-6	6	-6	-4	6	-6	6	-6	6	-6	6	-6
$q[K_1]$	0	2	1	1	2	0	1	2	2	1	2	2	1	1
$q[K_2]$	0	2	1	1	2	0	1	2	2	1	1	1	2	2
$q[K_3]$	0	2	1	2	1	0	1	2	1	2	1	2	1	2
$q^Y[K_1]$	2	0	0	0	0	2	0	0	0	0	0	0	0	0
$q^Y[K_2]$	2	0	0	0	0	2	0	0	0	0	0	0	0	0
$q^Y[K_3]$	2	0	0	0	0	2	0	0	0	0	0	0	0	0
class	✓	☺		–		✓	☺		–					

Table 7: Phenomenologically viable L_3 models with a $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ subgroup. Six of them, indicated by the symbols ✓ and ☺, fall into a class where already the discrete $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ symmetry drastically limits the allowed $\mathcal{B}$ and $\mathcal{L}$ violating operators, see table 6.

- **IX–XIV:** $\mathcal{Q} = \mathcal{K}_3$. In these cases, the existence of a conserved quantity $\mathcal{Q}$ does not guarantee a stable proton. We must resort to the full list of baryon and lepton number violating operators up to dimension five. It shows that, in all models, the only baryon number violating operators are $QQQH_1$ and $QQD^{c\dagger}$. Lepton number, on the other hand, is violated through $SN^cN^cN^c$ in all models, and additionally through

$$
\begin{aligned}
&N^c K_1^\dagger K_2\,,\ N^c K_1^c K_2^{c\dagger}\,,\ S N^c K_1^c K_2 &&\text{in XI and XII}\,,\\
&N^c K_1^{c\dagger} K_2^c\,,\ N^c K_1 K_2^\dagger\,,\ S N^c K_1 K_2^c &&\text{in XIII and XIV}\ .
\end{aligned}
$$

Since baryon and lepton number are separately violated only at the non-renormalizable level, the proton is safe in these models.

Having presented phenomenologically viable $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ models which are symmetric under L_3, we now discuss the shortcomings of two anomaly-free L_3 symmetric $\mathrm{U}(1)'$ charge assignments which lead to contradictions with observations. Except for the exotic quarks, the $\mathrm{U}(1)'$ charges in both cases are identical to those of models I–V. The first case has

$$
\Big(z[K_1], z[K_2], z[K_3]\Big) = (1,2,8)\,, \qquad \Big(y[K_1], y[K_2], y[K_3]\Big) = (0,0,6)\,,
$$

leading to no conserved quantity $\mathcal{Q}$. Up to dimension five, $U^cD^cD^c$ is the only baryon number violating operator; lepton number is violated in

$$
E^c K_1 K_3^c\,,\ N^c K_1 K_2^c\,,\ N^c N^c K_1^c K_2\ .
$$

From the latter two terms one can obtain the effective operator $N^cN^cN^c$ at the loop level by contracting K_i with K_i^c, $i = 1, 2$. Therefore the diagram leading to proton decay is suppressed by only one power of M in this case.

In the second example, the exotic quarks have charges

$$
\Big(z[K_1], z[K_2], z[K_3]\Big) = (3,6,6)\,, \qquad \Big(y[K_1], y[K_2], y[K_3]\Big) = (2,4,4)\ .
$$

This choice results in the conserved quantity $\mathcal{Q} = \alpha\mathcal{L} + \beta\mathcal{K}_1$. As lepton number is conserved, one might consider this a physically acceptable charge assignment. However, the exotic quarks K_2 and K_3 mix with the up-type quarks through the superpotential operators (for the sake of clarity we suppress all generational indices)

$$
M\,KU^c\,, \qquad \frac{1}{M}\,SH_2QK^c\ . \tag{7.2}
$$

After S and H_2 acquire their vevs, we obtain the mass terms

$$
\begin{pmatrix} U & K \end{pmatrix} \cdot \begin{pmatrix} c_{11}\,\langle H_2\rangle & c_{12}\,\frac{\langle S\rangle\langle H_2\rangle}{M} \\ c_{21}\,M & c_{22}\,\langle S\rangle \end{pmatrix} \cdot \begin{pmatrix} U^c \\ K^c \end{pmatrix}, \tag{7.3}
$$

with eigenvalues of the order

$$
c_{21}\,M\,, \qquad \left(\frac{c_{11}c_{22}}{c_{21}} - c_{12}\right) \cdot \frac{\langle S\rangle\langle H_2\rangle}{M}\ . \tag{7.4}
$$

Assuming no artificially small value for the coupling coefficient c_{21}, the second mass eigenvalue is way too small to account for the up-type quark masses. Therefore, a scenario in which the exotic quarks mix with the observed ones as in eq. (7.2) would be highly unnatural.

8. Models with $\mathbb{Z}_3^Y \times \mathbb{Z}_{N>3}$

We can now relax the requirement of integer $\mathrm{U}(1)'$ charges for the exotic quarks. After rescaling the charges, this is tantamount to looking for scenarios where $\mathrm{U}(1)' \to \mathbb{Z}_{N>3}$. Indeed, we find many such anomaly-free models. For B_3 and L_3, some are given in ref. [17]. It is the purpose of this section to argue for the stability of the proton in the models of ref. [17], as well as in some new models featuring the discrete symmetry M_3. Concerning the B_3 models of ref. [17], we have already shown in section 4 that the proton is absolutely stable. The $\mathrm{U}(1)'$ charge assignments which we are going to discuss here are only the L_3 and M_3 cases given in table 8. The primed models are related to the unprimed ones by simultaneously changing $y[K_i] \leftrightarrow y[K_i^c]$ and $z[K_i] \leftrightarrow z[K_i^c]$; the thus obtained charge assignments are also anomaly-free because the anomaly coefficients do not distinguish between $\mathrm{SU}(3)_C$ triplets and antitriplets (see ref. [17]).

In order to determine whether a model is consistent with the longevity of the proton, we take the same approach as in the previous section. First we filter out the information contained in the $\mathbb{Z}_3^Y \times \mathbb{Z}_N$ subgroup. If the rank $\mathfrak{r}$ of the set of homogeneous linear equations derived from the allowed operators, see eq. (5.3), is less than 5, we have to find the conserved quantity $\mathcal{Q}$ for these scenarios. In some cases, no further effort has to be made because the discrete symmetry already stabilizes the proton. However, often we have to take a second step and determine the conserved quantity $\mathcal{Q}$ of the specific model (i.e. using the exact $\mathrm{U}(1)'$ charges). If that also fails, we need to investigate explicitly all baryon and lepton number violating operators up to dimension five.

L_3 models. With only the discrete symmetry $\mathbb{Z}_3^Y \times \mathbb{Z}_N$ at hand, the rank $\mathfrak{r}$ reduces only in cases (ii)/(ii') of table 8; we get the conserved quantity $\mathcal{Q} = \mathcal{K}_1 - 2\mathcal{K}_3$. Even if we disregard the operators $N^cN^cN^c$ and $SN^cN^cN^c$, $\mathfrak{r}$ is not reduced in the other six cases. For (ii)/(ii'), at the level of the discrete symmetry, we obtain $\widetilde{\mathcal{Q}} = \alpha(3\mathcal{L} \mp \mathcal{K}_2) + \beta(\mathcal{K}_1 - 2\mathcal{K}_3)$, where the upper sign holds true for the unprimed model and the lower for the primed one. We stick to this convention throughout this section. Since the actual charges of models (ii)/(ii') allow neither $N^cN^cN^c$ nor $SN^cN^cN^c$, lepton number is conserved in all processes without external exotic quarks. So models (ii)/(ii') are phenomenologically acceptable.

For the remaining six L_3 cases, we need to consider the exact $\mathrm{U}(1)'$ charge assignments. The obtained conserved quantities for the models are:

$$\begin{array}{lll} & \text{including } (S)N^cN^cN^c & \text{excluding } (S)N^cN^cN^c \\ \text{(i)/(i'):} & \mathcal{Q} = 3\mathcal{L} \mp (\mathcal{K}_1 - 2\mathcal{K}_2 - 2\mathcal{K}_3)\,, & \widetilde{\mathcal{Q}} = 3\mathcal{L} \mp (\mathcal{K}_1 - 2\mathcal{K}_2 - 2\mathcal{K}_3)\,, \\ \text{(iii)/(iii'):} & \mathcal{Q} = \beta\mathcal{K}_1 + \gamma(\mathcal{K}_2 + \mathcal{K}_3)\,, & \widetilde{\mathcal{Q}} = \alpha(\mathcal{L} \pm \mathcal{K}_3) + \beta\mathcal{K}_1 + \gamma(\mathcal{K}_2 + \mathcal{K}_3)\,, \\ \text{(iv)/(iv'):} & \mathcal{Q} = \mathcal{K}_1 - 2\mathcal{K}_2 - 2\mathcal{K}_3\,, & \widetilde{\mathcal{Q}} = \alpha\mathcal{L} + \beta(\mathcal{K}_1 - 2\mathcal{K}_2 - 2\mathcal{K}_3)\;. \end{array}$$

In models (i)/(i'), the proton is safe due to $\mathcal{L}$ conservation in processes with no external K_i. The other models violate $\mathcal{L}$.

However, for models (iv)/(iv') the only lepton number violation occurs in $SN^cN^cN^c$. We must therefore determine the baryon number violating operators. It is worth pointing out that already at the level of the discrete $\mathbb{Z}_3^Y \times Z_N$ symmetry, in all eight L_3 cases the

	L_3 models								M_3 models				
$z[Q]$	0		0		0				0			0	
$z[U^c]$	-18		-90		-9				6			15	
$z[D^c]$	9		45		0				3			-6	
$z[L]$	-3		-15		-3				3			3	
$z[N^c]$	-6		-30		3				-6			3	
$z[E^c]$	12		60		3				0			-9	
$z[H_1]$	-9		-45		0				-3			6	
$z[H_2]$	18		90		9				-6			-15	
$z[S]$	-9		-45		-9				9			9	
	(i)	(i')	(ii)	(ii')	(iii)	(iii')	(iv)	(iv')	(v)	(vi)	(vii)	(viii)	(ix)
in ref. [16]	BV-II	BV-II'	BV-III	BV-III'	BV-V	BV-V'	BV-VI	BV-VI'					
$z[K_1]$	13	-4	47	-2	7	2	5	4	-5	-5	-5	-11	-11
$z[K_2]$	-8	17	-40	85	1	8	-1	10	-5	-2	-2	7	-13
$z[K_3]$	-8	17	-49	94	-2	11	-1	10	-8	-2	-7	-13	-16
$y[K_1]$	2	-2	2	-2	2	-2	2	-2	0	2	2	2	2
$y[K_2]$	-4	4	-4	4	-4	4	-4	4	0	-4	-4	-4	4
$y[K_3]$	-4	4	-4	4	-4	4	-4	4	6	-4	4	4	4
$q^Y[K_1]$	2	1	2	1	2	1	2	1	0	2	2	2	2
$q^Y[K_2]$	2	1	2	1	2	1	2	1	0	2	2	2	1
$q^Y[K_2]$	2	1	2	1	2	1	2	1	0	2	1	1	1

Table 8: Some models with $\mathrm{U}(1)' \to \mathbb{Z}_{N>3}$.

only BV operators are precisely the ☺ operators from section 6:

$$U^c D^c D^c \,, \quad S U^c D^c D^c \,, \quad Q Q Q H_1 \,, \quad Q Q D^{c\dagger} \,. \tag{8.1}$$

The specific charges of models (iv)/(iv') forbid the first two terms, so both $\mathcal{L}$ and $\mathcal{B}$ are only violated separately at the non-renormalizable level. Hence, these models have a sufficiently stable proton.

Concerning models (iii)/(iii'), we must additionally determine all lepton number violating operators up to dimension five:

$$\begin{array}{lll} & \mathcal{B} \text{ violation} & \mathcal{L} \text{ violation} \\ \text{(iii):} & QQQH_1\,,\, QQD^{c\dagger}\,, & N^c K_2^\dagger K_3\,,\, N^c K_2^c K_3^{c\dagger}\,,\, SN^c K_2^c K_3\,,\, SN^c N^c N^c\,, \\ \text{(iii'):} & QQQH_1\,,\, QQD^{c\dagger}\,, & N^c K_2^{c\dagger} K_3^c\,,\, N^c K_2 K_3^\dagger\,,\, SN^c K_2 K_3^c\,,\, SN^c N^c N^c\,. \end{array}$$

We see that in models (iii)/(iii') baryon and lepton number are violated separately only at the non-renormalizable level. So these are also viable charge assignments.

***M_3* models.** The symmetry M_3 forbids baryon number violation among the MSSM particles and the right-handed neutrinos. For models (v) and (vi), the discrete hypercharges satisfy eq. (6.1), which anticipates that $\mathcal{B}$ is also conserved at the level of the subgroup $\mathbb{Z}_3^Y \times \mathbb{Z}_9$ once we add the exotic quarks. For the remaining three cases (vii)−(ix), the discrete symmetry of the models also leads to the conserved quantity $\mathcal{Q} = \mathcal{B}$. Therefore, all five M_3 models given in table 8 have a stable proton.

9. Summary and conclusion

In this article, we investigated the issue of proton stability in the general UMSSM with R-parity violation. The proton decay problem may arise due to two reasons. First, in the absence of R-parity, one might expect the usual RPV couplings to destabilize the proton. However, the LV-BV separation [17] ensures that the dangerous LV and BV couplings cannot coexist, so that the proton is safe from operators involving MSSM fields, even at the non-renormalizable level. The second, much more severe problem arises due to the presence of light exotics, which are needed to render the $\mathrm{U}(1)'$ gauge symmetry free of anomalies. The exotics themselves may have LV and/or BV interactions, posing a serious problem for the stability of the proton. Nevertheless, we have identified several classes of models where the exotics are relatively harmless with respect to the proton decay issue.

A central element in our analysis was the concept of discrete gauge symmetries. After the spontaneous breaking of the $\mathrm{U}(1)'$ gauge symmetry, any charge assignment automatically leads to a remnant $\mathbb{Z}_N$ symmetry. Furthermore, there is an analogous $\mathbb{Z}_N^Y$ discrete symmetry which is left over after the breaking of the hypercharge gauge group $\mathrm{U}(1)_Y$. We found that the knowledge of these discrete symmetries provides a powerful tool in arguing for the stability of the proton. Our main results are pictorially summarized in figure 1, where we present the main steps one has to follow in deciding whether a particular UMSSM model is safe with respect to proton decay or not. We should stress that figure 1 can be applied only to anomaly-free UMSSM models with a minimal exotic content, i.e. three generations of $\mathrm{SU}(2)_L$-singlet exotic quarks. Our method, however, can be easily generalized to the case of non-minimal exotic sectors as well.

In section 4 we identified four symmetries (B_3, P_6, $R_{12}L_{12}^4$, $R_{12}^5L_{12}^8$) which render the proton absolutely stable. Figure 1 confirms that the shortest path to the stable proton outcome is when the model exhibits a B_3 discrete symmetry. For other discrete symmetries, knowing the couplings of the exotic quarks to the MSSM particles and the right-handed neutrinos is essential. For this reason, we have extended the concept of a discrete symmetry to the exotic sector. Since the hypercharge of the new exotic particles is also unknown, we introduced the notion of a discrete hypercharge. Scanning all possible $\mathbb{Z}_3^Y \times \mathbb{Z}_3$ extensions of lepton triality L_3 and matter triality M_3, we found many cases in which the discrete symmetry forbids (most of) the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L$ invariant baryon and/or lepton number violating operators up to dimension five. Their absence for a particular $\mathrm{U}(1)'$ charge assignment ensures a stable proton. This is confirmed by figure 1, which offers several alternative paths to the stable proton outcome, which rely primarily on the extended discrete symmetries encoded in the model.

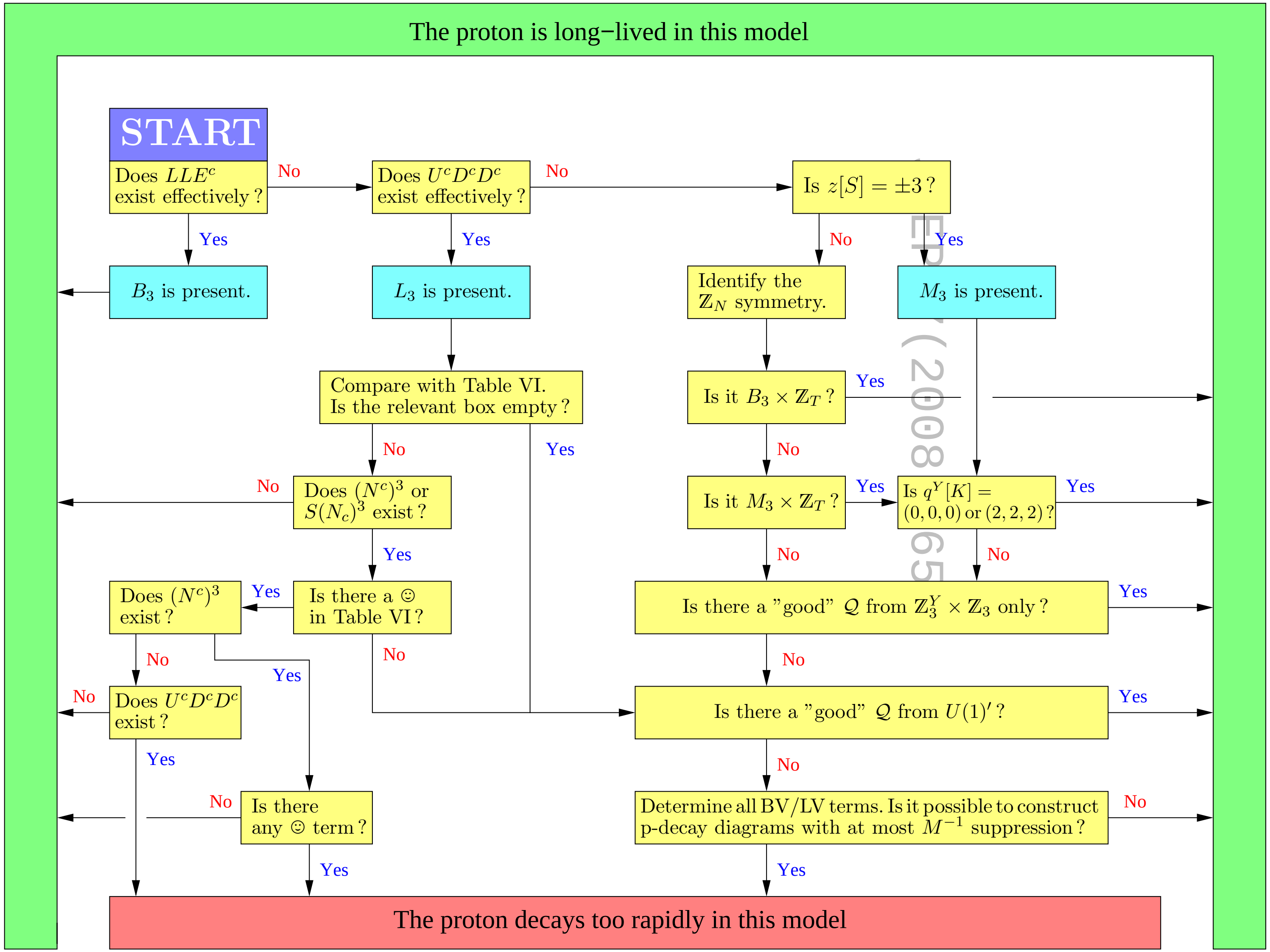

Figure 1: A roadmap for RPV UMSSM model building.

Models which do not fall into these "good" categories need to be further scrutinized. A method which we found very useful in classifying the remaining possibilities is the following. We generate all possible operators up to dimension five which are invariant under either $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'$ or its discrete version $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathbb{Z}_3^Y \times \mathbb{Z}_N$. We then look for a conserved quantity $\mathcal{Q}$ among the set of those operators. As the proton cannot decay into exotic quarks, one can often argue for a sufficiently stable proton solely on the basis of $\mathcal{Q}$. As evidenced from figure 1 and some of our examples in sections 6–8, this can be often the case with L_3 and M_3 UMSSM models. Only for a few remaining model cases, it is necessary to explicitly write down all baryon and lepton number violating operators in order to verify whether the proton is stable.

Our results show that in spite of the presence of light exotics at the TeV scale, the anomaly-free RPV UMSSM is a phenomenologically viable alternative to more conventional versions of low energy supersymmetry. It is instructive that a consistent model has *three* new elements in comparison to the (N)MSSM: (1) new $\mathrm{U}(1)'$ gauge interactions and the associated gauge particles and their superpartners; (2) RPV interactions and (3) new exotic isosinglet quarks and squarks at the TeV scale [32]. One should therefore be on the lookout for such signatures during the upcoming runs at the Large Hadron Collider at CERN.

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A. Good $\mathbb{Z}_3^Y \times \mathbb{Z}_6$ extensions

We have pointed out in section 6 that those extensions of the $\mathbb{Z}_6$ symmetries $R_6 L_6^2$ ($\cong M_3 \times R_2$), and $R_6^3 L_6^2$ ($\cong L_3 \times R_2$) which have the discrete hypercharges of eq. (6.1) conserve baryon number up to dimension five operators. The complete $\mathbb{Z}_3^Y \times \mathbb{Z}_6$ scan reveals that no additional good symmetries that comply with eq. (6.8) are obtained for $R_6 L_6^2$.

The situation changes dramatically when scanning over the extensions of the symmetry $R_6^3 L_6^2$. Out of the $27 \times 56 = 1512$ possible discrete charge assignments,[11] 1298 have rank $\mathfrak{r}$ smaller than 5, leading to a conserved quantity $\mathcal{Q}$. Filtering out those cases which satisfy eq. (6.8), we are left with 415 cases with non-trivial $\mathcal{Q}$. For illustration, we list those 16 scenarios in which the rank is reduced to $\mathfrak{r} = 2$, together with the corresponding conserved quantity $\mathcal{Q}$, in table 9. α, β, γ are free real parameters. Therefore, one actually has three independent conserved quantities in these scenarios.

[11]Concerning the $\mathbb{Z}_6$ sector, there are 6 cases with identical $q[K_i]$, $6 \times 5 = 30$ cases where two $q[K_i]$ are identical, and finally $\frac{6 \cdot 5 \cdot 4}{3!} = 20$ cases with all three discrete charges different from each other. This adds up to 56 different $\mathbb{Z}_6$ charge assignments for the exotic quarks.

$(q^Y[K_1], q^Y[K_2], q^Y[K_3])$	$(q[K_1], q[K_2], q[K_3])$	$\mathcal{Q}$
$(0,0,0)$ or $(2,2,2)$	$(1,1,1)$ or $(3,3,3)$ or $(5,5,5)$	$\alpha\mathcal{B} + \beta\mathcal{L} + \gamma(\mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3)$
$(1,1,2)$	$(3,3,3)$ or $(3,3,5)$	$\alpha(\mathcal{B} + \mathcal{K}_1 + \mathcal{K}_2) + \beta\mathcal{L} + \gamma\mathcal{K}_3$
$(1,2,1)$	$(3,3,3)$	$\alpha(\mathcal{B} + \mathcal{K}_1 + \mathcal{K}_3) + \beta\mathcal{L} + \gamma\mathcal{K}_2$
$(2,1,1)$	$(3,3,3)$ or $(1,3,3)$	$\alpha(\mathcal{B} + \mathcal{K}_2 + \mathcal{K}_3) + \beta\mathcal{L} + \gamma\mathcal{K}_1$
$(1,2,2)$	$(3,3,3)$ or $(3,5,5)$	$\alpha(\mathcal{B} + \mathcal{K}_1) + \beta\mathcal{L} + \gamma(\mathcal{K}_2 + \mathcal{K}_3)$
$(2,1,2)$	$(3,3,3)$	$\alpha(\mathcal{B} + \mathcal{K}_2) + \beta\mathcal{L} + \gamma(\mathcal{K}_1 + \mathcal{K}_3)$
$(2,2,1)$	$(3,3,3)$ or $(1,1,3)$	$\alpha(\mathcal{B} + \mathcal{K}_3) + \beta\mathcal{L} + \gamma(\mathcal{K}_1 + \mathcal{K}_2)$

Table 9: $\mathbb{Z}_3^Y \times \mathbb{Z}_6$ extensions of $R_6^3 L_6^2$ which lead to three conserved quantities $\mathcal{Q}$.

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